

# Systems and Software Verification

## Chapter 8. Liveness Properties

Ver. 2.0

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# 8. Liveness Properties

- Liveness property
  - Under certain conditions, some event will ultimately occur.
  - Some happy event will occur in the end.
  - Examples:
    - (L1) " Any request will ultimately be satisfied "
    - (L2) " By keeping on trying, one will eventually succeed "
    - (L3) " If we call on the elevator, it will bound to arrive eventually "
    - (L4) " The light will turn green (some day regardless of the system behavior)"
    - (L5) " After the rain, the sunshine "
    - (L6) " The program will terminate "
  - Two broad family of liveness properties
    1. Simple liveness : *progress* (Chapter 8)
    2. Repeated liveness : *fairness* (Chapter 10)
- Organization of Chapter 8
  - Simple Liveness in Temporal Logic
  - Are Liveness Properties Useful?
  - Liveness in the Model, Liveness in the Properties
  - Verification under Liveness Hypotheses
  - Bounded Liveness

# 8.1 Simple Liveness in Temporal Logic

- $F \phi$ 
  - “  $\phi$  will ultimately occur. ”
  - (L1) “ Any request will ultimately be satisfied ”
    - $AG(\text{req} \Rightarrow AF \text{sat})$
  - (L7) “ The system can always return to its initial state ”
    - $AG EF \text{init}$
  - $P U Q$ 
    - “ Along the execution, we will find a state satisfying Q and P will hold for all the states encountered in the meantime ”
    - Regarded as a liveness property
    - $P U Q \equiv F Q \wedge (P W Q)$   
(liveness) (safety)
    - $A(PUQ)$  and  $E(PUQ)$  are all liveness properties.

## 8.2 Are Liveness Properties Useful?

- Abstract liveness properties
  - “ If we call on the elevator, it is bound to arrive eventually ”
    - It yields no information, from a utilitarian viewpoint.
    - “Abstract” liveness property
  - “ An event will occur within at most  $x$  time unit ”
    - It is useful, but became a safety property.
    - “Bounded” liveness property
  - But, it is still useful
    - “Abstract” more general than “concrete”
    - “Abstract” more efficient than “concrete”
    - “Abstract” and “concrete” are not contradictory

## 8.3 Liveness in the Model, Liveness in the Properties

- Two different roles in the verification process
  1. Liveness *properties* : we wish to verify
  2. Liveness *hypotheses* : we make on the system model
- When we use a mathematical model(automata) to represent a real system,
  - The semantics of the model in face define *implicit safety and liveness hypotheses*.
  - Safety hypothesis :
    - Clear
    - It can flip from  $q$  to  $q'$  only if it includes a transition going from  $q$  to  $q'$ .
  - Liveness hypothesis :
    - Not clear
    - The system will chain transitions as long as possible. (to a block state or accepting states)
    - " The system does not terminate without reason, or remain inactive indefinitely without reason. "
    - Can be subtle and cause errors :



- One must be aware of the premises of the models used and check their adequacy !

## 8.4 Verification under Liveness Hypotheses

- Verify that specific model behaviors satisfy a given property :
  - $\phi_v$  : only the model which the liveness hypotheses hold
  - $\psi$  : a property
  - Verify  $\phi_v \Rightarrow \psi$  is sufficient!!!
  - If  $\psi$  is a CTL property
    - $AF ( E P U Q ) \rightarrow A ( \phi_v \Rightarrow FE ( \phi_v \wedge P U Q ) )$

# 8.5 Bounded Liveness

- Bounded liveness property
  - A liveness property that comes with a maximal delay which the desired situation must occur.
  - Safety properties from a theoretical viewpoint.
  - Can be rewritten in a form  $AG (\psi_2 \Rightarrow F^{-1} \psi_1)$
  - Not as important as safety properties
  
- Bounded liveness in timed systems
  - Often used in the specification of timed systems (in Chapter 5)
  - Explicit constraints on delays  $\rightarrow$  TCTL !!!
  
  - (BL1) " The program terminates in less than ten seconds "
    - $AF_{<10s} \text{ end}$   $\leftarrow$  bounded liveness property
    - $AG (\neg \text{end} \Rightarrow F^{-1}_{<10s} \text{ start} )$   $\leftarrow$  safety property
  
  - (BL2) " Any request is satisfied in less than five minutes "
    - $AG ( \text{req} \Rightarrow AF_{<5m} \text{ sat} )$   $\leftarrow$  bounded liveness property
    - $AG ( \neg(F^{-1}_{=5m} \text{req} \wedge G^{-1}_{\leq 5m} \neg \text{sat} )$   $\leftarrow$  safety property